High-Accuracy Integrated GPS-INS Aircraft Navigation for Landing using Pseudolites and Double-Difference Carrier Phase measurements

presented by:

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Overview

1. Introduction
2. Problem Description
3. GPS Overview
4. Precise Positioning using carrier phase measurements
5. Aircraft Landing Using Integrity Beacons
6. Single-Difference Carrier Phase Measurements
7. Integer Ambiguity Resolution
8. Double-Difference Carrier Phase Measurements
9. Least-Squares Ambiguity Decorrelation Adjustment (LAMBDA)
10. Simulation Results
11. Conclusion
12. Future Work
Introduction

- The Global navigation satellite systems are fast becoming preferred means of navigation.
- Satellite based navigation system allows to fly aircraft through time and fuel efficient routes.
- Provides efficient air traffic management.
- The commissioning and maintenance cost of the ground based signalling systems such as instrument landing system can be reduced by relying on GPS-based landing approach.
- Government of India has invested several thousand crores in building space infrastructure in the country. With GAGAN and IRNSS (Indian Regional Navigation Satellite System), air navigation in India will soon become more accurate and efficient.

Reference: http://www.gps.gov/applications/aviation/
Problem Description

- Provide precise navigation to an aircraft during landing and use PVT estimation to control the aircraft

The research work includes the following

- Simulation of an aircraft landing trajectory, presently it is kinematic
- Integer ambiguity resolution by achieving geometric diversity using integrity beacons
- Implementation of navigation algorithm using single-difference and double-difference carrier phase measurements
- Development and simulation of the integrated GPS-INS navigation system
- Development of the flight control system
- Receiver autonomous integrity monitoring
GPS Overview

- GPS Segments: Space Segment, Control Segment and User Segment
- Signals: Each satellite transmits two RF signals L1 and L2
  \[ f_{L1} = 1575.42 \text{ MHz}, \quad \text{and} \quad f_{L2} = 1227.60 \text{ MHz} \]

GPS measurements

- Code phase measurement (Pseudo-range measurement)
  \[ \rho = r_{true} + l_{\rho(t)} + T_{\rho(t)} + c(\delta t_u - \delta t^s) + \xi_{\rho} \]  

- Carrier phase measurement
  \[ \phi = \lambda^{-1}[r - l_{\Phi} + T_{\Phi}] + \frac{c}{\lambda}(\delta t_u - \delta t^s) + N + \xi_{\Phi} \]  

Where: N is unknown no. of whole cycles, the carrier phase measurement has no information about the no. of whole cycles; this is also called integer ambiguity.

\[ \sigma(\xi_{\rho}) = 0.5 \text{ m}, \sigma(\xi_{\Phi}) = 5 \text{ mm} \]
GPS Overview

Errors in measurement
- Ionospheric and tropospheric refraction (5 m)
- Clock and Ephemeris Errors (3 m)
- Multipath Errors (1.5 m)
- Receiver noise (0.6 m)
- Satellite Geometry

**Figure:** GDOP

Reference:

Precise Positioning using carrier phase measurements

- The Pseudo-range measurements provide accuracy around 5-20 meters.
- This accuracy level is not acceptable in some applications such as aircraft landing.
- Centimetre level accuracy can be achieved by using differential carrier phase measurements.
- However, carrier phase measurements require estimation of unknown integer ambiguities.
Aircraft Landing Using Integrity Beacons

- One integrity beacon is placed on each side of the approach path to a runway.
- Ground reference station measures the carrier phase of the GPS and Integrity Beacon signals.
- These measurements are transmitted to the aircraft.
- The on-board system consists a GPS receiver, a data link receiver and a computer to execute the navigation algorithm.

Figure: Aircraft landing using Integrity Beacon

Aircraft Landing Using Integrity Beacons (contd..)

Figure: Aircraft landing system

Single-Difference Carrier Phase Measurements

Carrier phase measurements at the user (Aircraft) from \( i^{th} \) satellite

\[
\phi_u^i = \lambda^{-1} [r_u^i - l_u^i + T_u^i] + c(\delta t_u - \delta t_s^i) + N_u^i + \epsilon_{\phi,u}^i
\]  

(3)

Single-Difference Carrier Phase Measurements (contd..)

- Carrier phase measurement at the reference station from the same satellite

\[ \phi^i_r = \lambda^{-1}[r^i_r - l^i_r + T^i_r] + c(\delta t_r - \delta t^i_s) + N^i_r + \epsilon^i_{\phi,r} \]  

(4)

Single-difference carrier phase

\[ \phi^i_{ur} = \phi^i_u - \phi^i_r \]

\[ \phi^i_{ur} = \lambda^{-1}[(r^i_u - r^i_r) - (l^i_u - l^i_r) + (T^i_u - T^i_r)] + c(\delta t_u - \delta t_r) + (N^i_u - N^i_r) + \epsilon^i_{\phi,u} - \epsilon^i_{\phi,r} \]

(5)

- Satellite clock error is eliminated with this differencing.
- Reference station and user are in proximity, so tropo. delay and iono. delay are eliminated.
- \( N_u - N_r \) is an unknown differential integer
- We need to estimate this differential integer
- The estimation process is called integer ambiguity resolution
Single-Difference Carrier Phase Measurements (Contd...)

Figure: Geometry of the single difference measurements

\[ r_{ur} = r_u - r_r \]  \hspace{1cm} (6)

\[ r_{ur}^i = -1_r^i \hat{X}_{ur} \]  \hspace{1cm} (7)

\(-1_r^i\) is the unit vector from ref. station to \(i^{th}\) satellite.

\[ \lambda \phi^i_{ur} = -1_r^i \hat{X}_{ur} + c \delta t_{ur} + \lambda \hat{N}_{ur}^i + \lambda \epsilon_{\phi,r}^i \]  \hspace{1cm} (8)

Carrier phase of the signal from integrity beacons $IB_j$ measured at the aircraft

$$\lambda \phi^j_u = | -p_j + X_{ur}| + c(\delta t_u - \delta t^j) + \lambda N^j_u + \lambda \epsilon_{\phi,u} \quad (9)$$

At the ref. station

$$\lambda \phi^j_r = |p_j| + c(\delta t_r - \delta t^j) + \lambda N^j_r + \lambda \epsilon_{\phi,r} \quad (10)$$

Single-difference carrier phase of the signal, received from the integrity beacons.

$$\lambda [\phi^j_u - \phi^j_r] = | -p_j + X_{ur}| - |p_j| + c(\delta t_u - \delta tr) + \lambda (N_u - N_r) + \lambda \epsilon_{\phi,u} - \lambda \epsilon_{\phi,r} \quad (11)$$
Integer Ambiguity Resolution

We define

\[ X_{ur} = \hat{X}_{ur} + X_{ur} \sim \quad (12) \]

- \( X_{ur} \) is the relative vector from user to reference station
- \( \hat{X}_{ur} \) is initial estimate and \( X_{ur} \sim \) is the correction to the estimate

The single-difference carrier phase equation for the \( i^{th} \) satellite can be written as

\[ \lambda \phi_{ur}^i + 1^i_r.\hat{X}_{ur} = -1^i_r.\overline{X}_{ur} + c\delta t_{ur} + \lambda N^i_{ur} + \lambda \epsilon_{\phi,ur}^i \quad (13) \]

The single-difference carrier phase equation for the integrity beacons can be written as

\[ \lambda \phi_{ur}^j - | - p_j + \hat{X}_{ur} | + | p_j | = - e^j . \overline{X}_{ur} + c\delta t_{ur} + \lambda N^j_{ur} + \lambda \epsilon_{\phi,ur}^j \quad (14) \]

the unit vector \( e^j = \frac{p_j - \hat{X}_{ur}}{|p_j - \hat{X}_{ur}|} \)
Integer Ambiguity Resolution (Contd...)

So, at time $t_k$ we have following measurements

\[
\lambda \delta \phi_{ur}^i(t_k) = -1_r^i X_{ur}(t_k) + \tau_k + \lambda N_{ur}^i + \lambda \epsilon_{\phi, ur}^i
\]  \hspace{1cm} (15)

\[
\lambda \delta \phi_{ur}^j(t_k) = -e^j X_{ur}(t_k) + \tau_k + \lambda N_{ur}^j + \lambda \epsilon_{\phi, ur}^j
\]  \hspace{1cm} (16)

\[-1^r \hat{X}_{ur} = \rho_{us} - \rho_{rs}\]

- $\lambda \delta \phi_{ur}(t_k), -1^i_r$ and $\lambda \epsilon_{\phi, ur}$ are known to us
- Our aim is to estimate $N_{ur}, \tau_k$ and correction $X_{ur}$
The unknowns in the eqs. for all GPS satellite and two Integrity Beacon are:

- $X_{ur}(t_k), \tau_k, N_{ur}^i$ and $N_{ur}^j$
  - $i=1,2,3...m$, $m$ is the no. of visible GPS satellites at time $t_k$
  - $j=1,2$ (Beacons)

To solve the above equations $N_{ur}^1$ is merged with $\tau_k$ and then we determine remaining $N_{ur}^i - N_{ur}^1$

\[
\begin{bmatrix}
\delta \phi_{ur}^1(t_k) \\
\delta \phi_{ur}^2(t_k) \\
\vdots \\
\delta \phi_{ur}^m(t_k) \\
\delta \phi_{IB1}^1(t_k) \\
\delta \phi_{IB2}^1(t_k)
\end{bmatrix} = \lambda
\begin{bmatrix}
-1^1_r(t_k) & 1 \\
-1^1_r(t_k) & 1 \\
-1^m_r(t_k) & 1 \\
-1^m_r(t_k) & 1 \\
-\epsilon^1_{IB1}(t_k) & 1 \\
-\epsilon^2_{IB1}(t_k)
\end{bmatrix}
\begin{bmatrix}
X_{ur}(t_k) \\
\tau_k
\end{bmatrix} + \lambda
\begin{bmatrix}
0 \\
N_{ur}^2 - N_{ur}^1 \\
\vdots \\
N_{ur}^m - N_{ur}^1 \\
N_{IB1} - N_{ur}^1 \\
N_{IB2} - N_{ur}^1 \\
\vdots \\
N_{IB1} - N_{ur}^1 \\
N_{IB2} - N_{ur}^1 \\
\vdots \\
\epsilon^1_{IB1} \\
\epsilon^2_{IB1} \\
\epsilon^m_{IB1} \\
\epsilon^1_{IB2} \\
\epsilon^2_{IB2} \\
\epsilon^m_{IB2}
\end{bmatrix}
\]

$(m+2) \times 1$ $(m+2) \times 4$ $(m+2) \times 1$ $(m+2) \times 1$
Aircraft collects the single diff. data during the bubble pass

The collected data is arranged in the following way

\[
\lambda \begin{bmatrix}
\delta \phi_1 \\
\delta \phi_2 \\
. \\
. \\
\delta \phi_n
\end{bmatrix}
= \begin{bmatrix}
\hat{S}_1 & 0 & 0 & . & I \\
0 & \hat{S}_2 & 0 & . & I \\
. & . & . & . & . \\
0 & 0 & 0 & \hat{S}_n & I
\end{bmatrix}
\begin{bmatrix}
X_1 \sim^* \\
X_2 \sim^* \\
. \\
. \\
X_n \sim^* \\
\ldots .... \\
N
\end{bmatrix}
+ \lambda \epsilon
\]

\[
l = \begin{bmatrix}
0 & 0 & . & 0 \\
1 & 0 & . & 0 \\
0 & 0 & . & 1
\end{bmatrix}
l: (m + 2) \times (m + 1)
\]

Above equation can be solved using least square technique

\( \hat{X}_{ur} \) is updated in each iteration

The above single difference formulation does not cause correlation in the vector \( \epsilon \)

Integer ambiguities are obtained by rounding the float solution to the nearest integer
Integer Ambiguity Resolution (Contd...)

\[ H = \lambda \begin{bmatrix} 
\delta \phi_1 \\
\delta \phi_2 \\
\vdots \\
\delta \phi_n 
\end{bmatrix}, \quad G = \begin{bmatrix} 
\hat{S}_1 & 0 & 0 & \cdots & 1 \\
0 & \hat{S}_2 & 0 & \cdots & 1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \hat{S}_n & 1
\end{bmatrix}, \quad Y = \begin{bmatrix} 
X_1^* \\
X_2^* \\
\vdots \\
X_n^* \\
\vdots \\
N
\end{bmatrix} \]

Size of matrices \( G, H, \) and \( Y \) is:

\[ G : n(m + 2) \times (4n + m + 1) \]

\[ H : n(m + 2) \times 1 \]

Least square estimation:

\[ Y = (G' \times G)^{-1} \times G' \times H \] (17)

- The \( G \) and \( H \) matrices are updated in each iteration
- Converges in 3-10 iteration
Table: Integer Ambiguities

<table>
<thead>
<tr>
<th>No.</th>
<th>True $N^K_{ur} - N^1_{ur}$</th>
<th>Estimated $N^K_{ur} - N^1_{ur}$</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-26093</td>
<td>-26092.99</td>
<td>0.0065</td>
</tr>
<tr>
<td>2</td>
<td>-62198</td>
<td>-62197.96</td>
<td>0.03</td>
</tr>
<tr>
<td>3</td>
<td>-118010</td>
<td>-118010.03</td>
<td>-0.02</td>
</tr>
<tr>
<td>4</td>
<td>-98252</td>
<td>-98252.063</td>
<td>-0.063</td>
</tr>
<tr>
<td>5</td>
<td>11727</td>
<td>11726.96</td>
<td>-0.032</td>
</tr>
<tr>
<td>6</td>
<td>-91595</td>
<td>-91595.09</td>
<td>-.09</td>
</tr>
<tr>
<td>7</td>
<td>26217</td>
<td>26216.98</td>
<td>-0.019</td>
</tr>
<tr>
<td>8</td>
<td>2624</td>
<td>2624.18</td>
<td>0.18</td>
</tr>
<tr>
<td>9</td>
<td>2624</td>
<td>2624.18</td>
<td>0.18</td>
</tr>
</tbody>
</table>

No of measurements: 60, $\sigma = 5$ mm
$\sigma$: Standard deviation of the noise
No. of visible satellites: 8
K=2,3,...8 (GPS satellite)
K=9,10 (Pseudolites)
Double-Difference Measurements

- The clock bias term $\delta t_{ur}$ is common in the single difference measurements from all satellites at each epoch. This term can be removed by the double-difference.

- Single-difference carrier phase measurement for satellite $k$ and $l$

$$\phi_{ur}^k = \phi_u^k - \phi_r^k$$

$$= \lambda^{-1} r_{ur}^k + f \cdot \delta t_{ur} + N_{ur}^k + \epsilon_{\phi,ur}^k \quad (18)$$

$$\phi_{ur}^l = \lambda^{-1} r_{ur}^l + f \cdot \delta t_{ur} + N_{ur}^l + \epsilon_{\phi,ur}^l \quad (19)$$

- Double-difference measurement

$$\phi_{ur}^{(kl)} = \phi_{ur}^k - \phi_{ur}^l$$

$$= \lambda^{-1} r_{ur}^{kl} + N_{ur}^{(kl)} + \epsilon_{\phi,ur}^{(kl)} \quad (21)$$

$$\phi_{ur}^{(kl)} = -\lambda^{-1} (1_r^k - 1_r^l) \cdot X_{ur} + N_{ur}^{(kl)} + \epsilon_{\phi,ur}^{(kl)} \quad (22)$$
If there are \( m \) visible satellites, there will be \( m-1 \) double difference equations.

If covariance of the carrier phase measurements is \( \rho_\phi^2 I \) then the covariance of a pair of double-difference measurements is

\[
\text{cov}_\phi(\dd) = 2\rho_\phi^2 \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}
\]

Double-difference measurements are correlated.

Simple rounding the float solution may give false results.
Decorrelation is achieved by using LAMBDA (Least-Squares Ambiguity Decorrelation Adjustment) technique

The estimation of unknown integers carried out in three steps: float solution, decorrelation, and fixed solution

\[ y = B\delta X + AN + \epsilon \]  \hspace{1cm} (25)

Cost function

\[ C_w = ||y - B\delta X - AN - \epsilon||^2_{Q_w^{-1}} \]  \hspace{1cm} (26)

*\(Q_w\): Covariance matrix of the double-difference measurements

*\(y\): Double-Difference measurements

Eq[26] is solved by using least square technique to obtain float solution
The float solution obtained from eq.[26]

$$\begin{bmatrix} \delta \hat{X} \\ \hat{N} \end{bmatrix}$$

(27)

$$Q_n = \text{Cov}(\hat{N})$$

(28)

where:

\(\hat{N}\) represents the float ambiguity vector

\(Q_n\) is the covariance matrix of the float solution \(\hat{N}\)

The second step consists of

$$\min ||\hat{N} - N||^2_{Q_n^{-1}}$$

(29)

- The ambiguities are decorrelated by a Z-transformation.
- The transformation matrix Z is determined by decomposing \(Q_n\) in a lower triangular matrix L and a diagonal matrix D

\[ \hat{M} = Z^T \hat{N} \]
\[ Q_{\hat{z}} = Z^T Q_{\hat{N}} Z \]

Where: \( \hat{M} \) is the transformed ambiguity vector
\( Q_{\hat{z}} \) is the transformed covariance matrix

The minimization process is performed on the transformed ambiguities

\[(\hat{M} - M)^T Q_{\hat{z}}^{-1}(\hat{M} - M) \leq \chi^2 \quad (30)\]

- The size of the search ellipsoid depends on the \( \chi \)
- The next step is

\[ N = Z^{-T} M \]
$Qhat = \begin{bmatrix} 4.97 & 3.87 \\ 3.87 & 3.01 \end{bmatrix}$ \hfill (31)

$\hat{N} = \begin{bmatrix} 0.119 \\ 0.157 \end{bmatrix}$ \hfill (32)

**Figure:** correlated

Reference: V., Sandra GNSS Research Centre, Curtin University Mathematical Geodesy and Positioning, Delft University of Technology
Decorrelated covariance matrix

\[ Q_{\text{zhat}} = \begin{bmatrix} 0.0865 & -0.0364 \\ -0.0364 & 0.0847 \end{bmatrix} \]  \hspace{1cm} (33)

\[ Z = \begin{bmatrix} -3 & -4 \\ 4 & 5 \end{bmatrix} \]  \hspace{1cm} (34)

\[ \hat{M} = \begin{bmatrix} 0.25 \\ 0.29 \end{bmatrix} \]  \hspace{1cm} (35)

**Figure:** Decorrelated
The ambiguities are correlated.

These are decorrelated by a $Z$ transformation.

$$\hat{M} = Z^T \hat{N}$$

$$Q_{z \text{hat}} = Z^T Q_{\text{hat}} Z$$
Figure: Decorrelated Ambiguities
Simulation and Results

Reference station position
Latitude: 0.5704 rad
Longitude: 1.2956 rad
Altitude: 314 m

Distance between integrity beacon and reference station: 16 km
Distance between integrity beacons: 4 km
Simulation and Results (contd...)

- Up to 60 Sec, pseudo-range measurements are used to determine the aircraft position after that it switches to the differential carrier phase mode.

![Position Error Graphs](image)

**Figure**: Position Error
Figure: Position Error
Simulation results show that centimetre level accuracy can be achieved with carrier phase measurements.

The flight trajectory is simulated without considering any forces and disturbances.

The integer ambiguity resolution requires that the user collect data from multiple epochs before a reliable estimate of the ambiguity can be made.
Future work

- Simulation of an aircraft landing trajectory
- An Integrated GPS-INS navigation system will be developed
- In the Integrated system INS solution will be updated by the GPS
- Development of the flight control system
References


Verhagen, S., and Li, B., *LAMBDA software package Matlab implementation*, version 3.0, Delft University of Technology, pp. 14

Thank you